Extreme perturbations in inflation

24 APRIL 2026, IBERICOS 2026, GRANADA

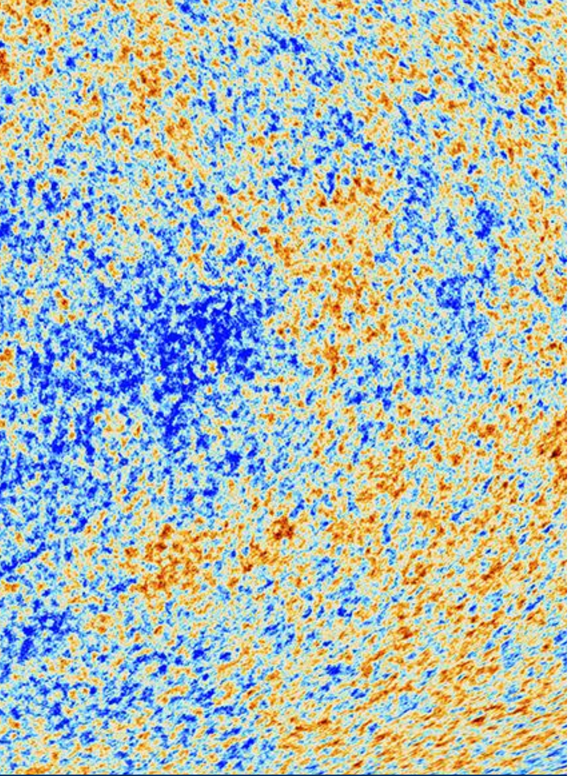
EEMELI TOMBERG

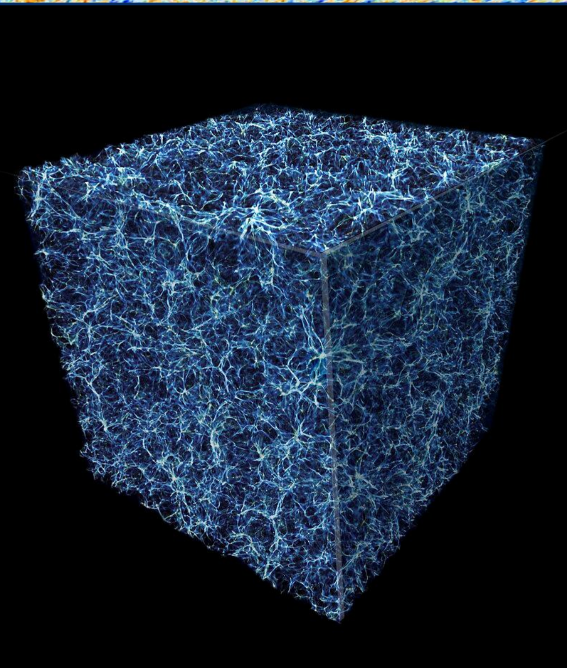
Extreme
cosmological perturbations

Stochastic inflation

Example: hilltop inflation

Recent developments





Cosmological perturbations

Inflation: linear perturbations

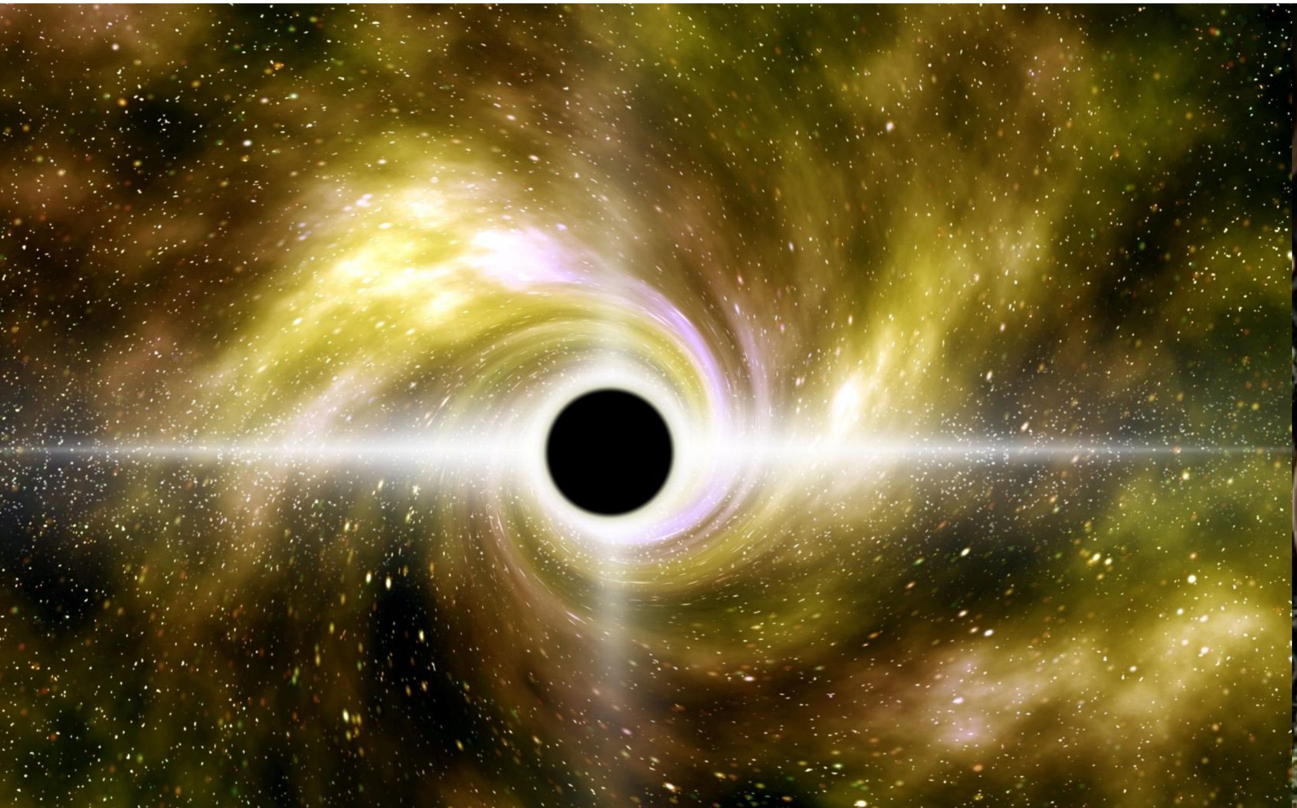
CMB

Growth of structure

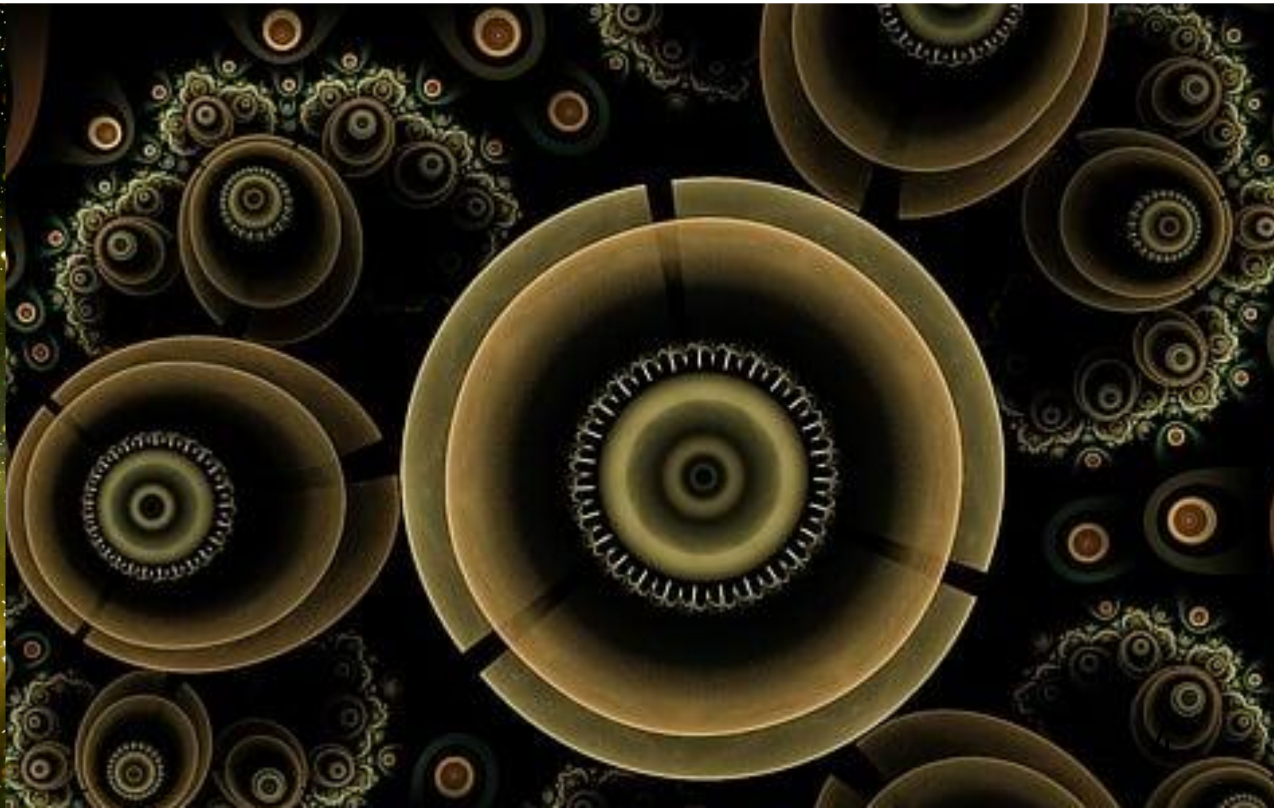
LSS

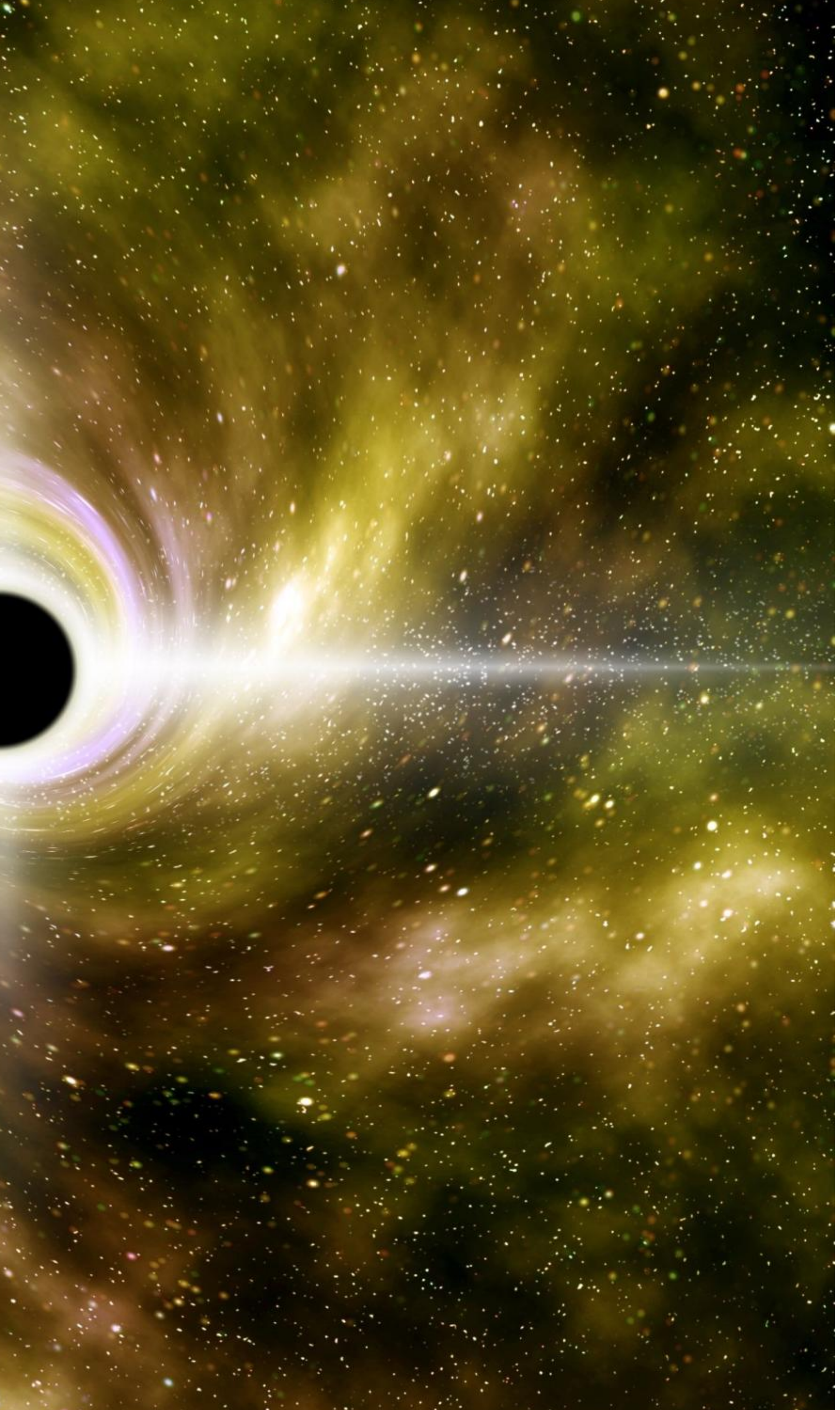
What about large primordial perturbations?

Primordial black holes



Eternal inflation





Primordial black holes

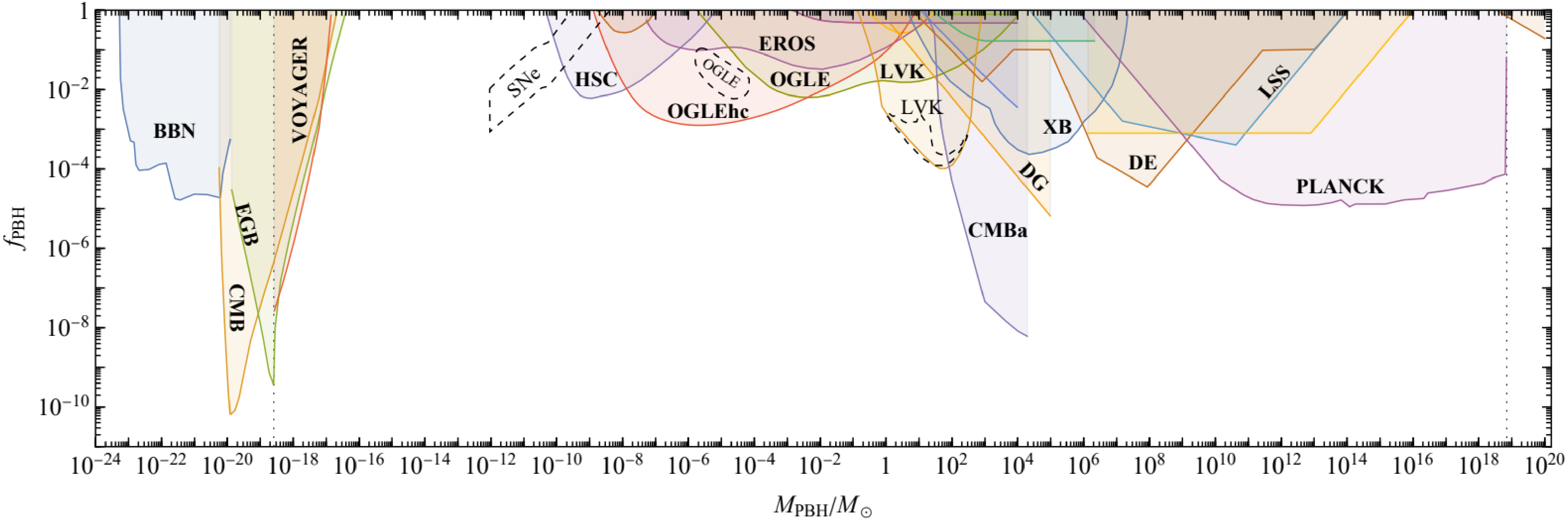
Collapse in the early Universe

Dark matter candidate

GW source

Seeds for supermassive BHs

Carr:2026hot

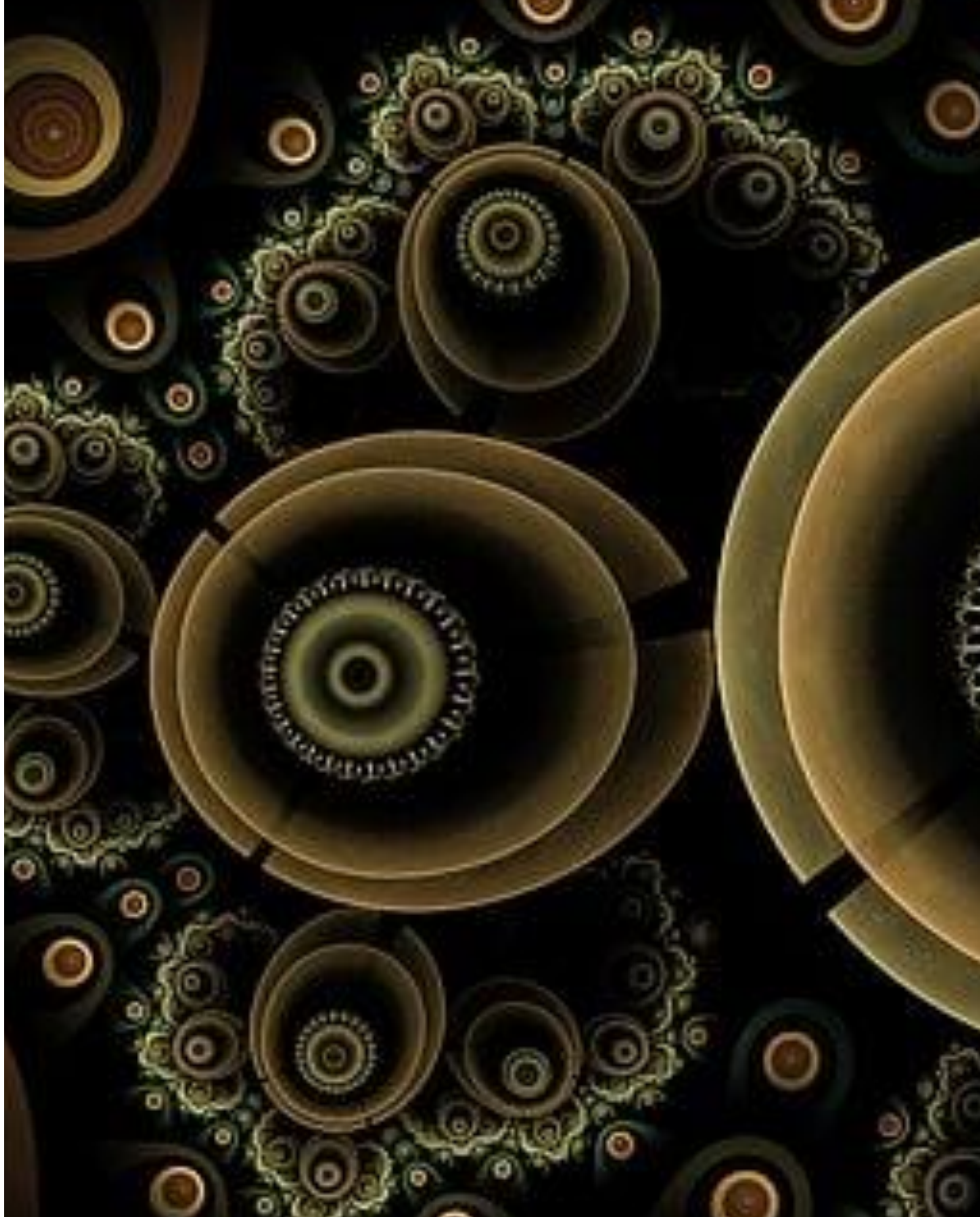


Rare perturbations make inflation last longer

Longer inflation: larger volume

Some inflating regions remain at all times

Eternal inflation



Properties of extreme perturbations

Abundance

Distribution in strength

Distribution in space

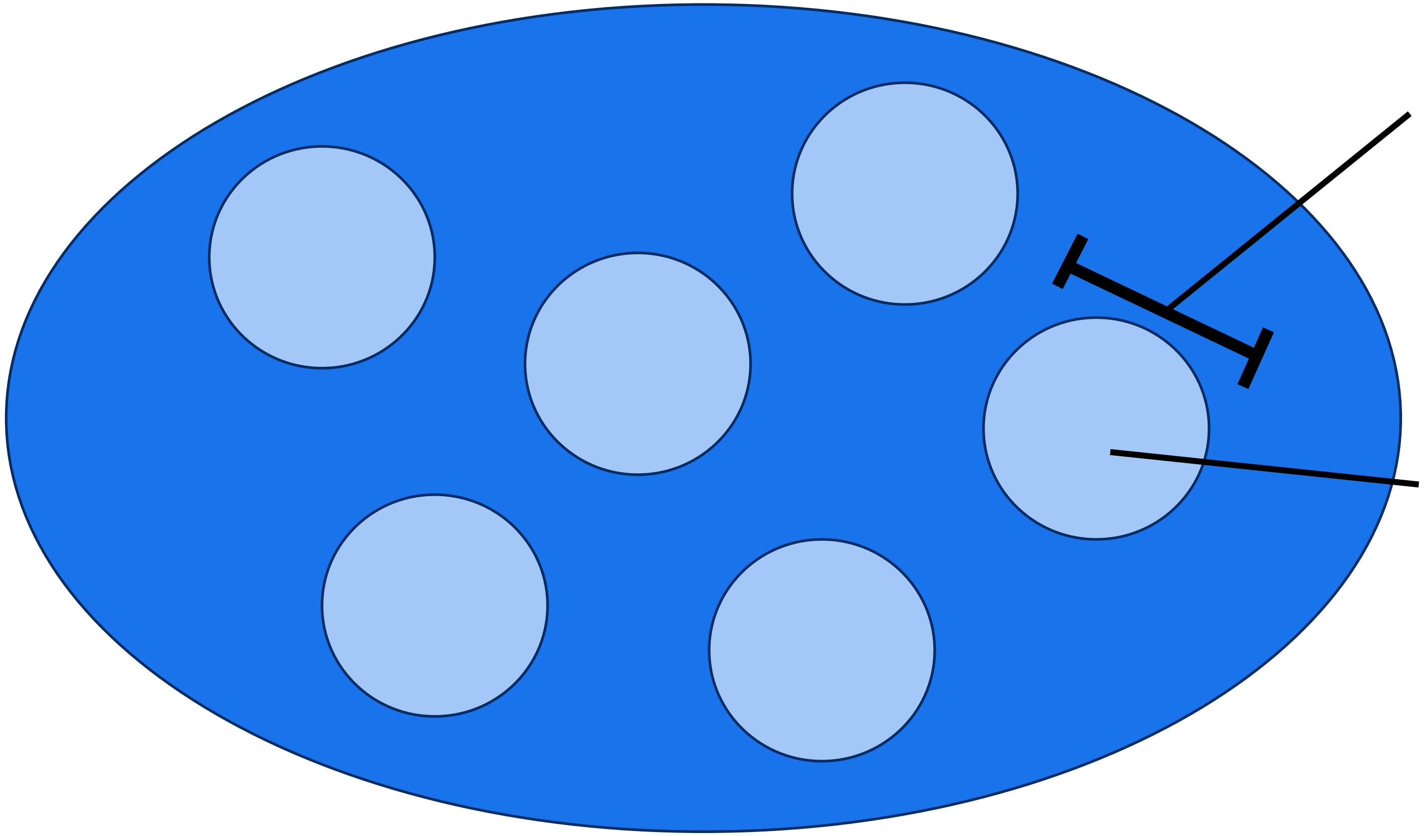
Tools

Lattice simulations of inflation

Stochastic inflation

ΔN formalism

ΔN formalism

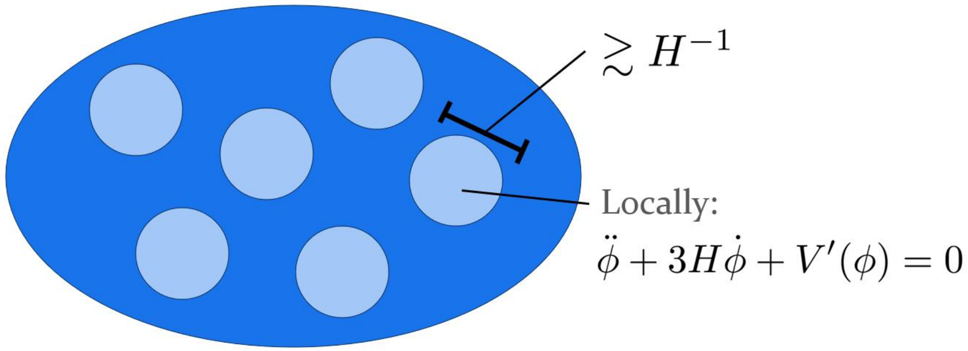


$\gtrsim H^{-1}$

Locally:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

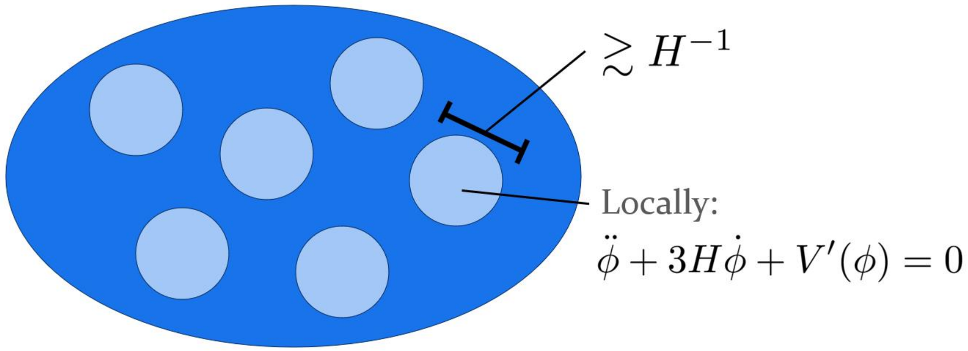
ΔN formalism



Separate universe approximation

Gradient expansion

ΔN formalism



$$
\begin{aligned}
\mathrm{d}s^2 &= -\mathrm{d}t^2 + a^2(t)e^{2\zeta(t,x)}\mathrm{d}x^2 \\
&= -\mathrm{d}t^2 + e^{2(\bar{N}(t)+\zeta(t,x))} \\
&= -\mathrm{d}t^2 + e^{2N(t,x)}\mathrm{d}x^2
\end{aligned}
$$

Separate universe approximation

$$\zeta = \Delta N = N - \bar{N}$$

Gradient expansion

Stochastic inflation

$$\phi_{\rm tot} = \phi + \delta\phi$$

$$\int_{k<k_\sigma} \frac{\mathrm{d}k^3}{(2\pi)^{2/3}} \phi_k(t) e^{-i\vec{k}\cdot\vec{x}}$$

Separate universe

$$\int_{k>k_\sigma} \frac{\mathrm{d}k^3}{(2\pi)^{2/3}} \phi_k(t) e^{-i\vec{k}\cdot\vec{x}}$$

Linear perturbations

Coarse-graining scale: $k = k_{\sigma_c} = \sigma_c a H$

Stochastic inflation

Slow-roll

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'(\phi)}{3H^2(\phi)} + \frac{H(\phi)}{2\pi}\xi(N)$$

$$\langle \xi(N)\xi(N') \rangle = \delta(N - N')$$

Stochastic inflation

Slow-roll

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'(\phi)}{3H^2(\phi)} + \frac{H(\phi)}{2\pi}\xi(N)$$

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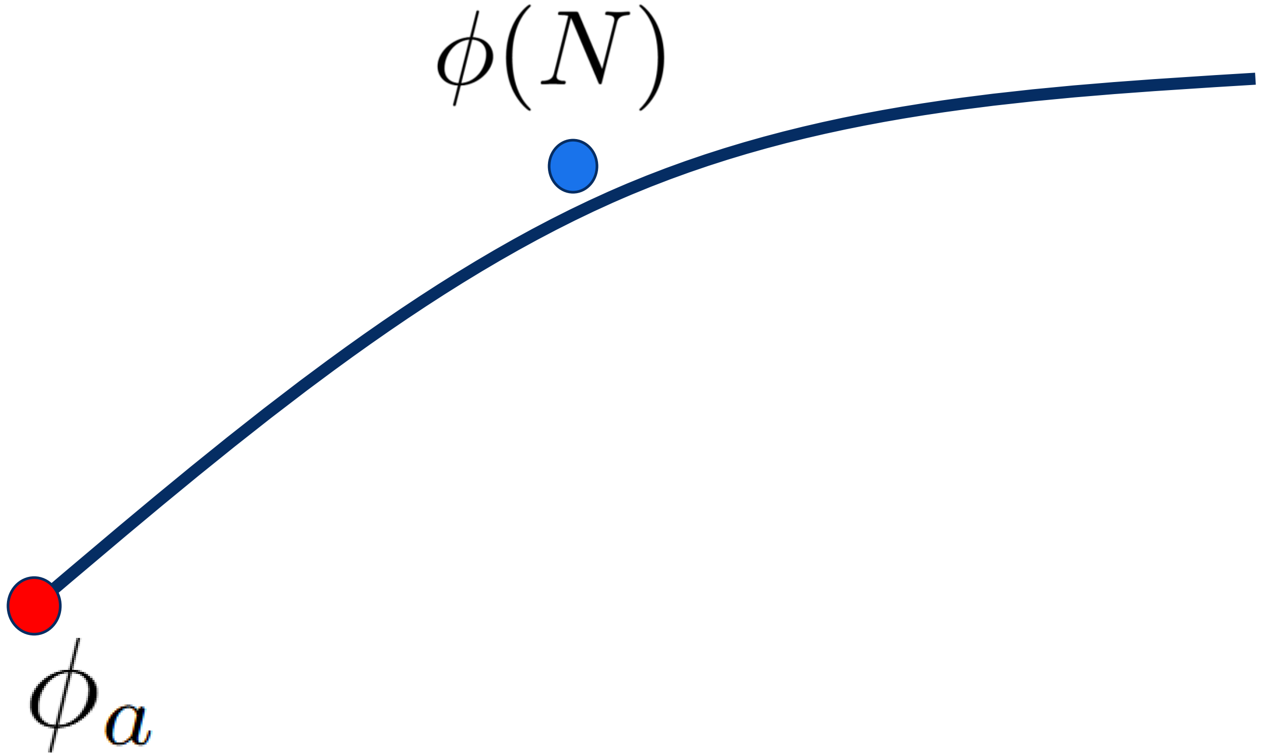
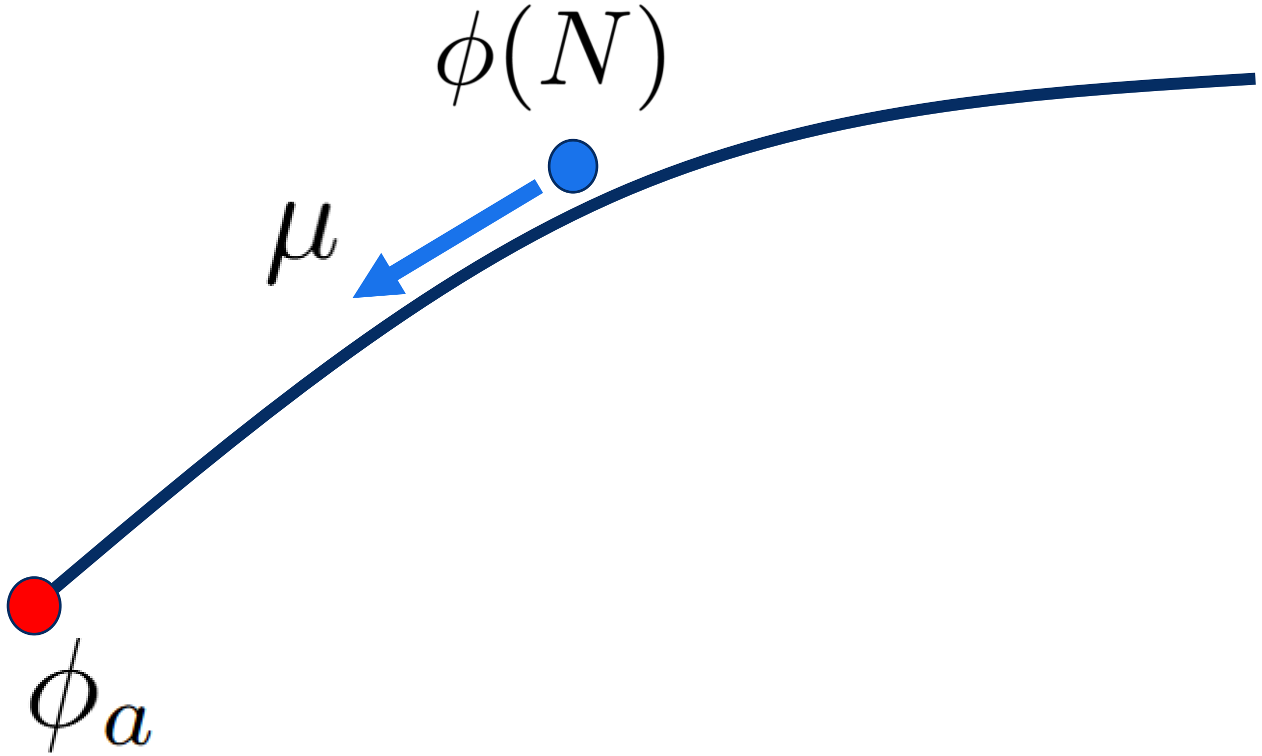
General attractor

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = \mu(\phi) + \sigma(\phi)\xi(N)$$

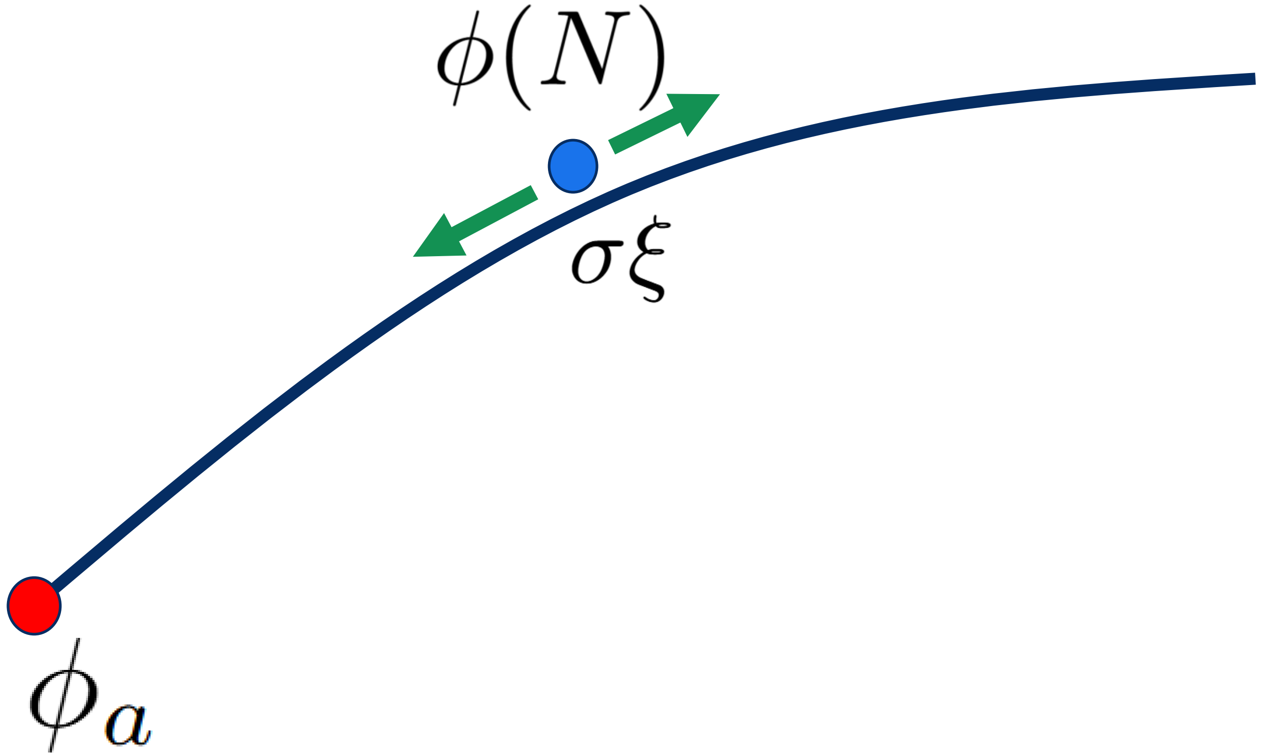
Stochastic inflation



Stochastic inflation



Stochastic inflation



Stochastic inflation

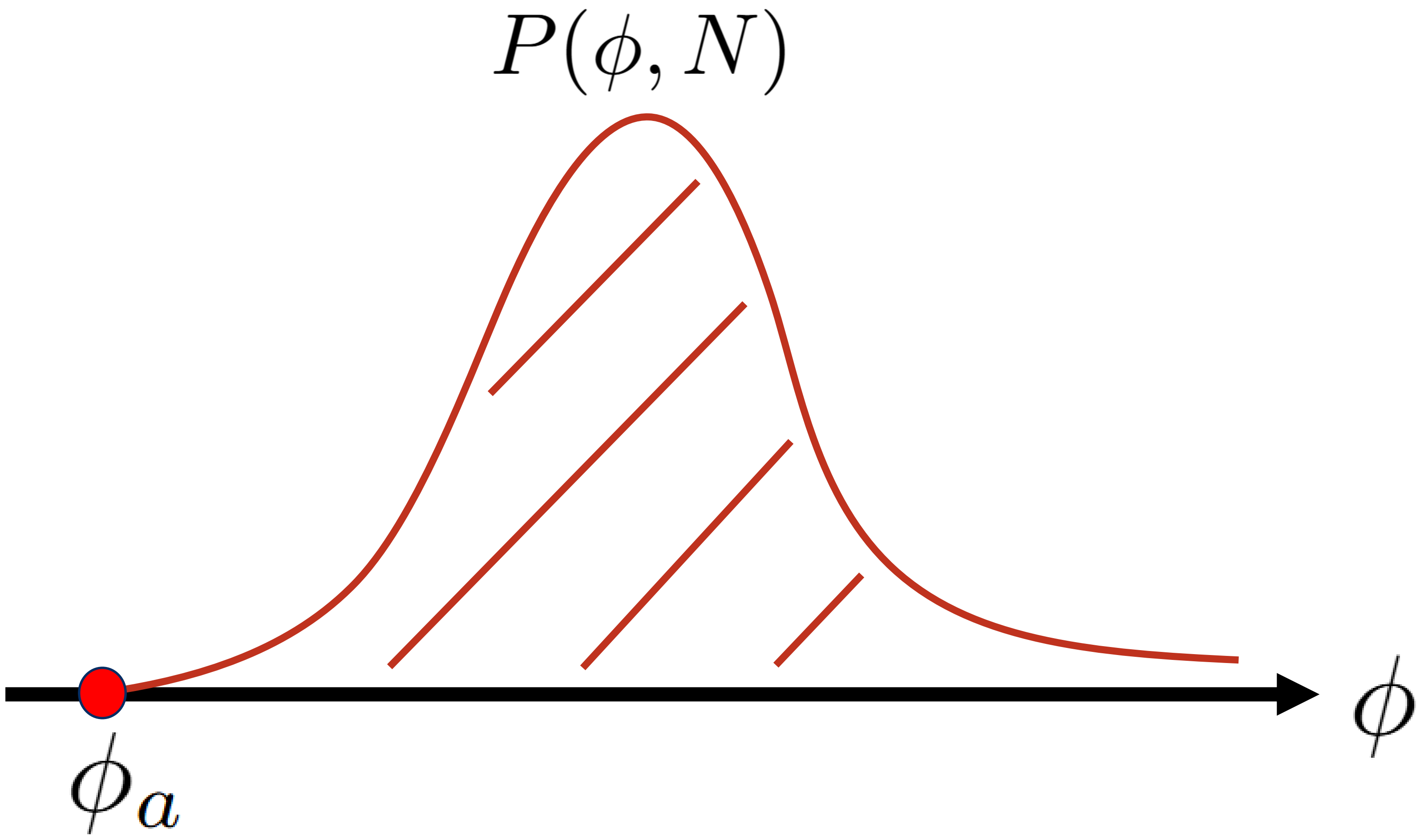
Fokker-Planck

$$\partial_N P(\phi, N) = \partial_\phi \left[\partial_\phi \left(\frac{1}{2} \sigma^2(\phi) P(\phi, N) \right) - \mu(\phi) P(\phi, N) \right]$$

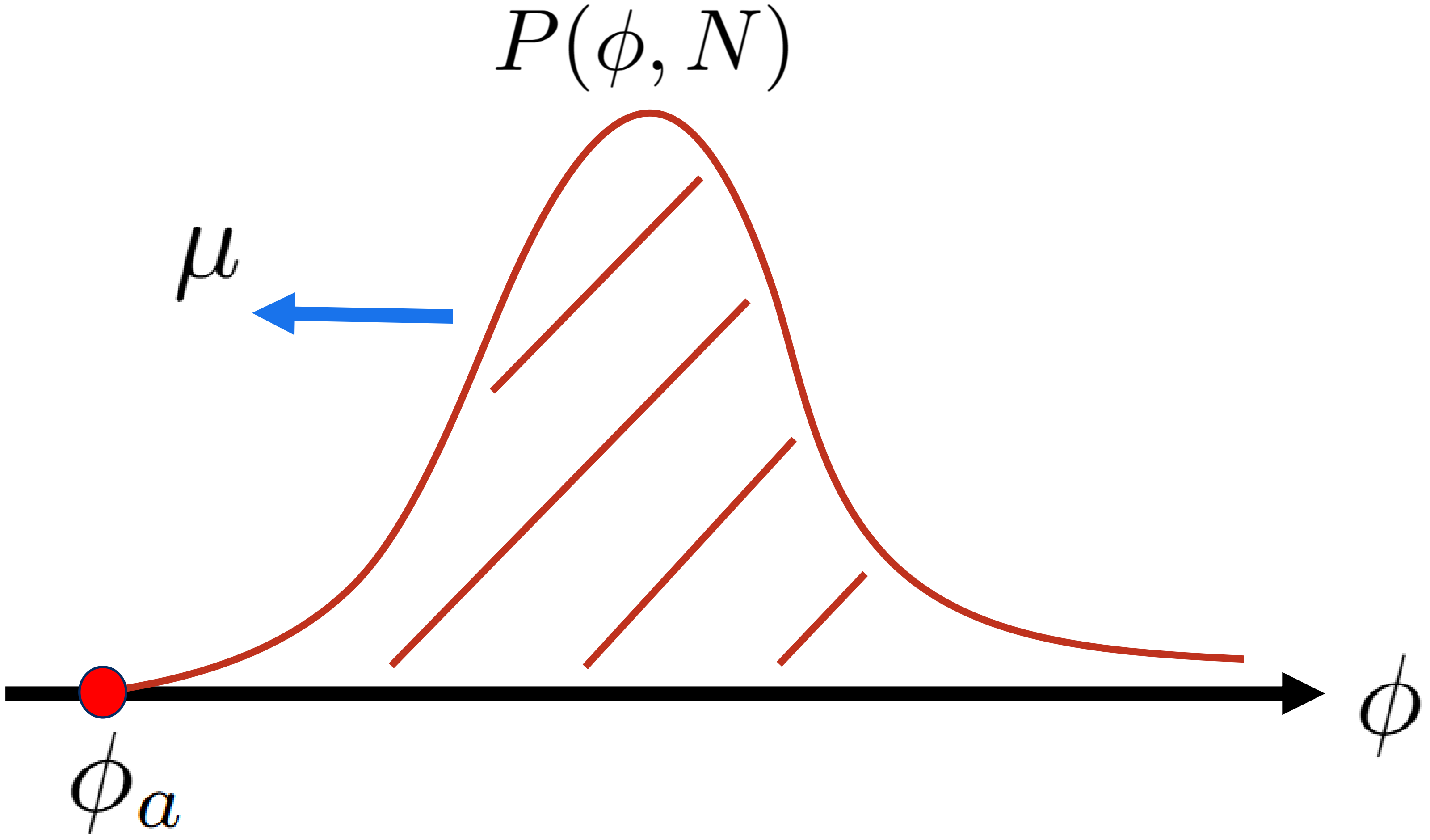
Adjoint Fokker-Planck

$$\partial_N P_{FPT}(N, \phi) = \frac{1}{2} \sigma^2(\phi) \partial_\phi^2 P_{\mathrm{FPT}}(\phi, N) + \mu(\phi) \partial_\phi P_{\mathrm{FPT}}(\phi, N)$$

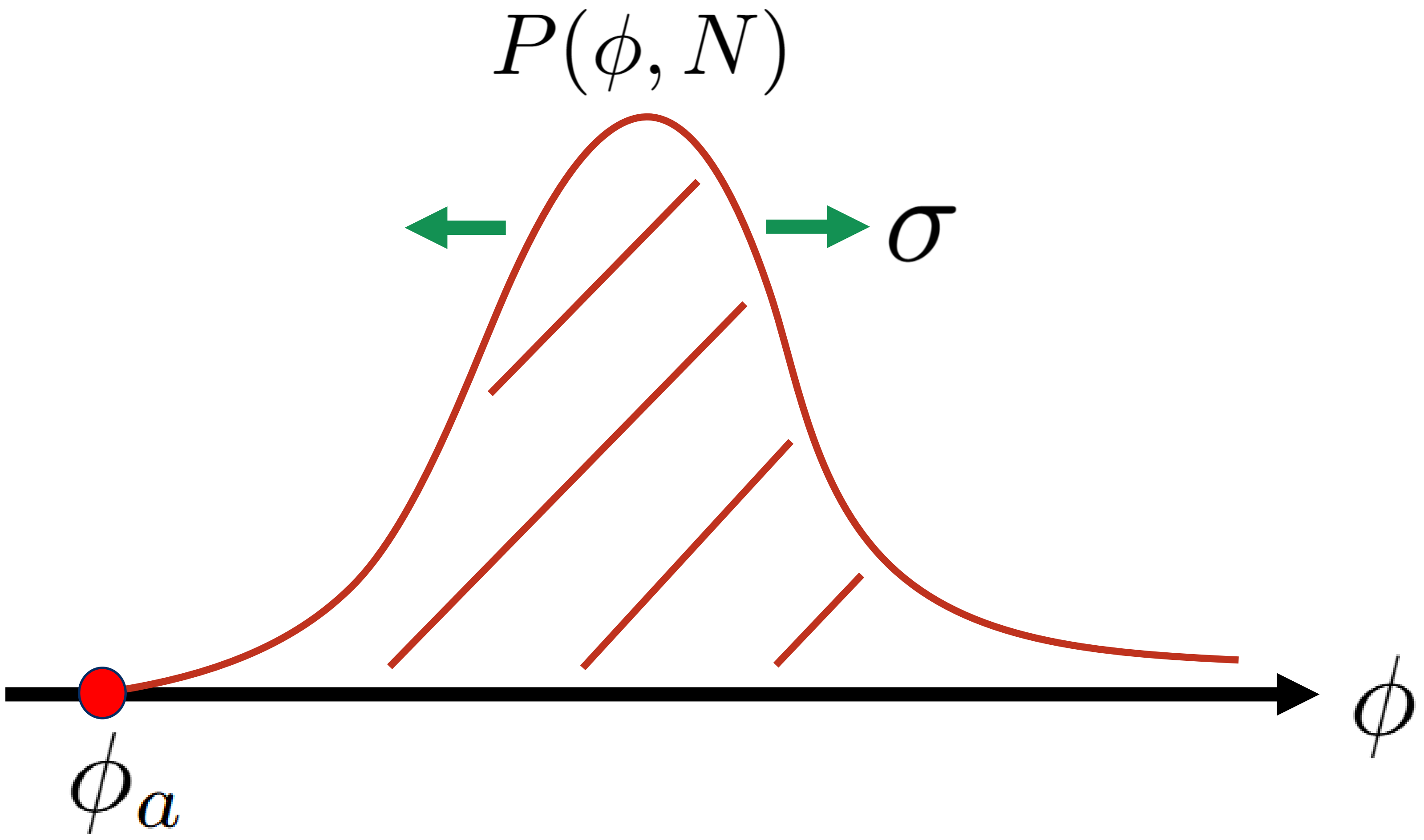
Stochastic inflation



Stochastic inflation



Stochastic inflation



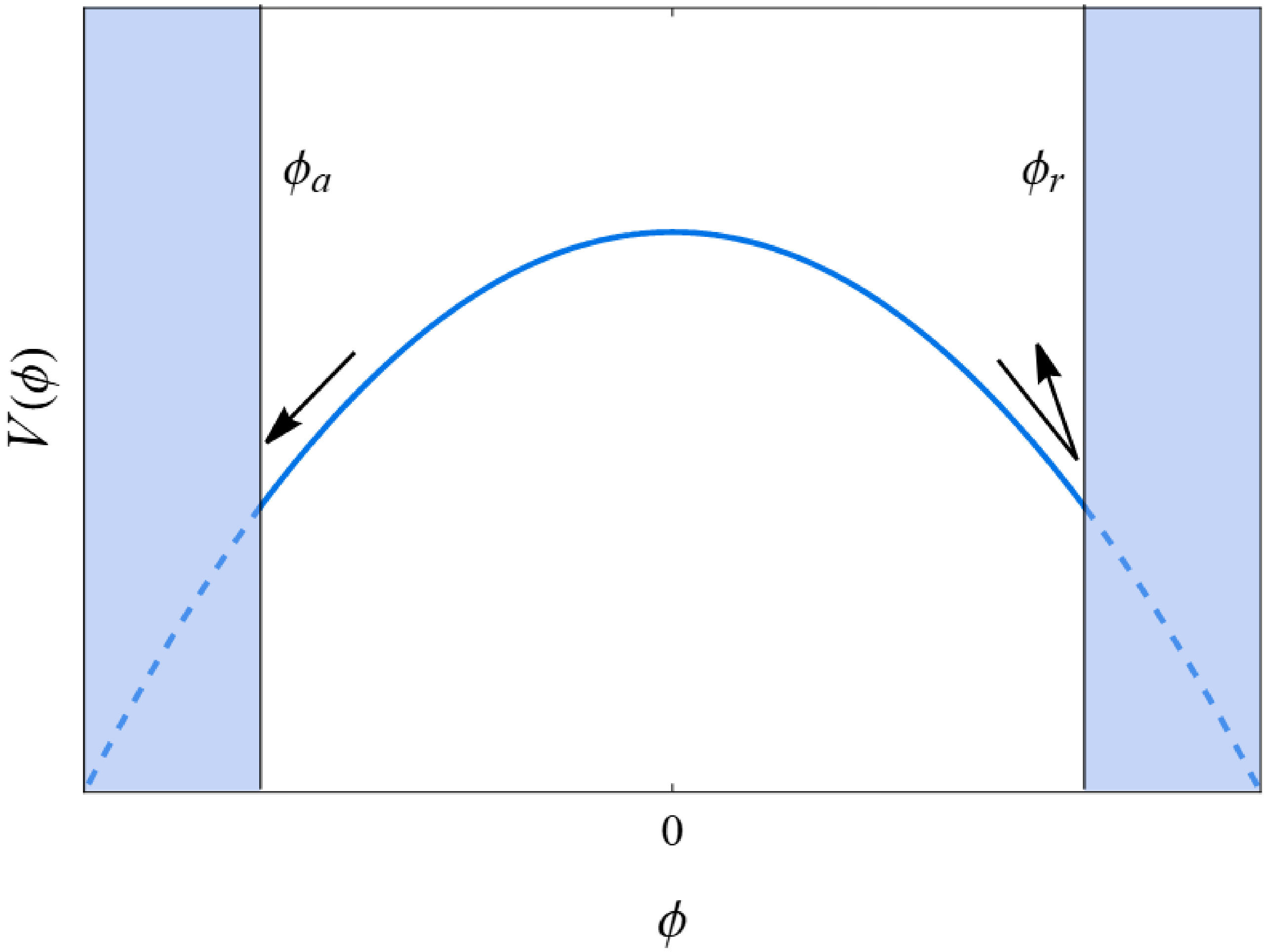
Hilltop inflation

Classical trajectory:

$$\phi_{\mathrm{cl}} = \phi_0 e^{\frac{\epsilon_2}{2} N}$$

$$\mu(\phi) = \frac{\epsilon_2}{2}\phi$$

$$\sigma = \frac{H}{2\pi}\frac{\sigma_c^{3/2}\sqrt{\pi}}{2}|H_\nu(\sigma_c)|$$



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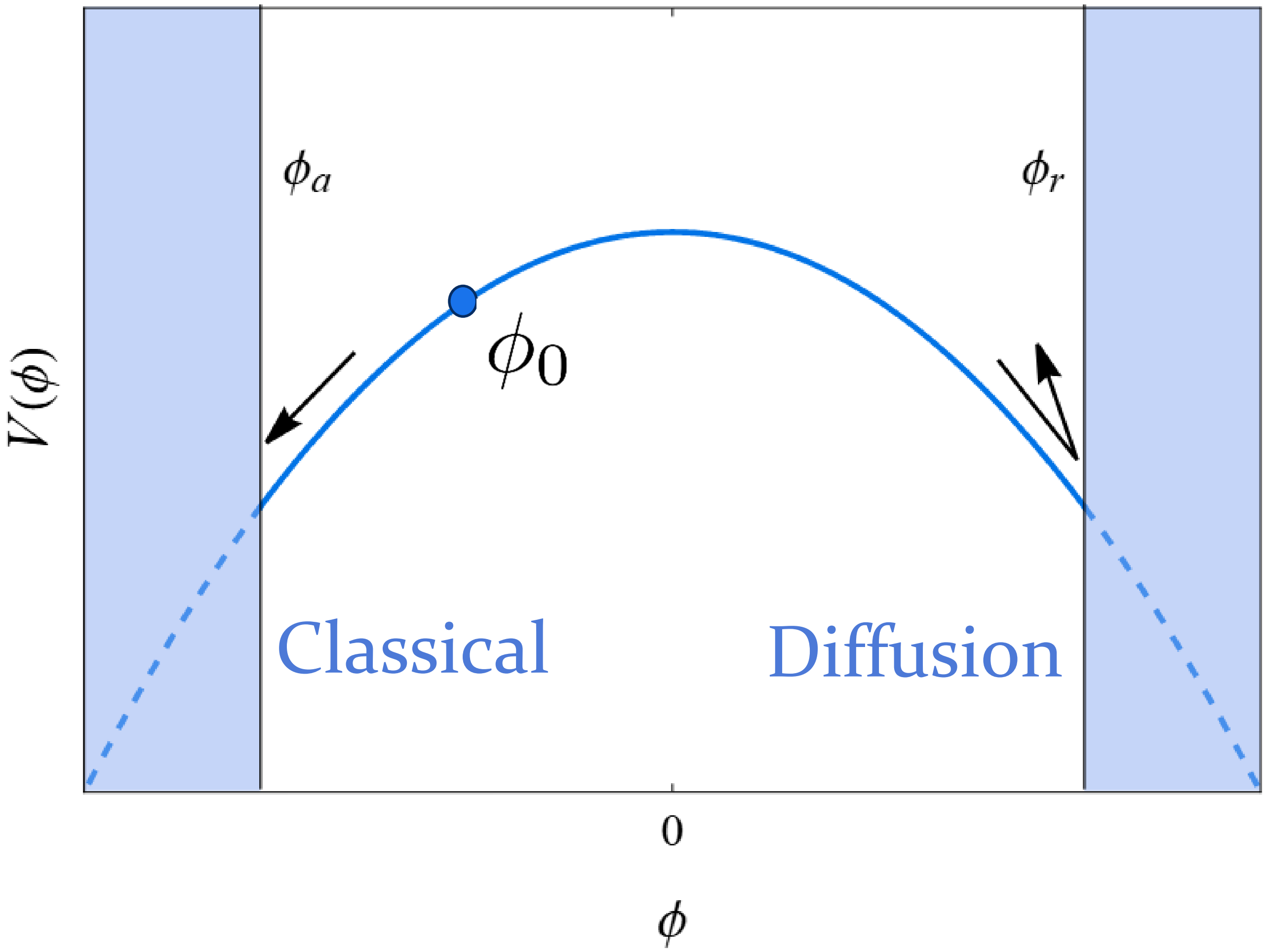


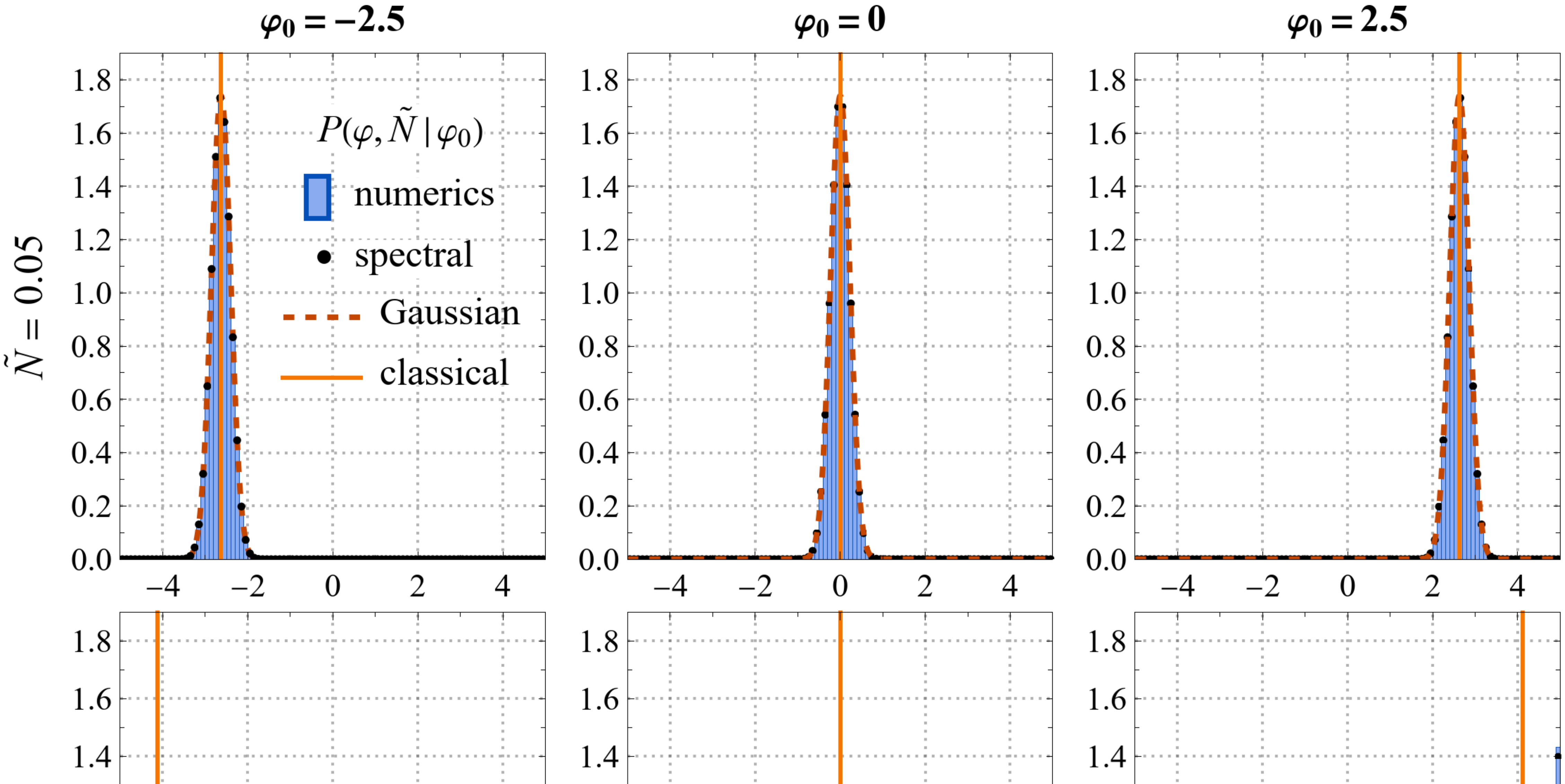
Hilltop inflation

Spectral decomposition:

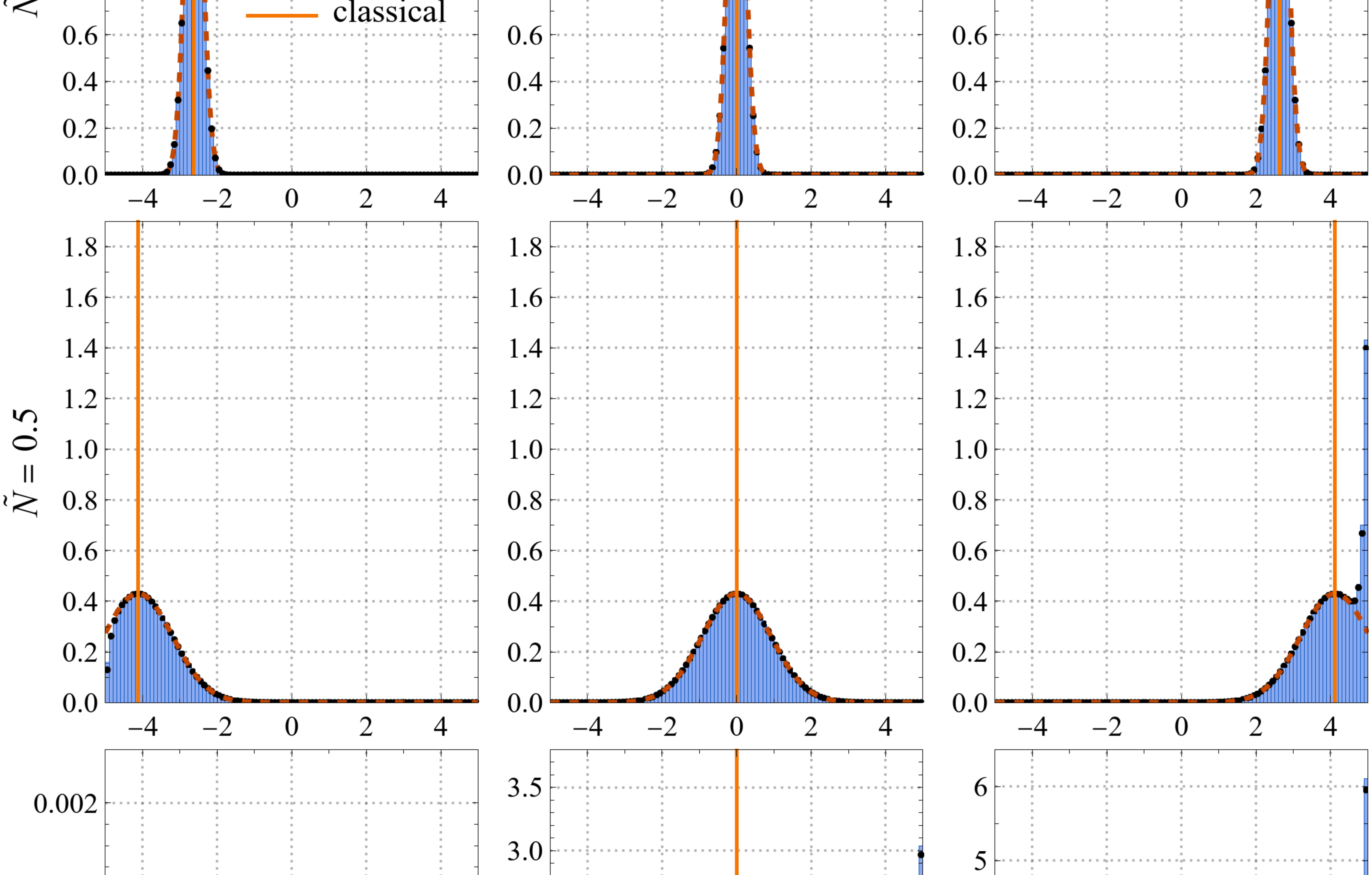
$$P(\phi, N) = \sum_{n=1}^{\infty} a_n u_n(\phi) e^{-\lambda_n N} \qquad P_{\mathrm{FPT}}(N, \phi) = \sum_{n=1}^{\infty} b_n \bar{u}_n(\phi) e^{-\lambda_n N}$$

$$\partial_\phi \left[\partial_\phi \left(\frac{1}{2} \sigma^2(\phi) u_n(\phi) \right) - \mu(\phi) u_n(\phi) \right] = -\lambda_n u_n(\phi)$$

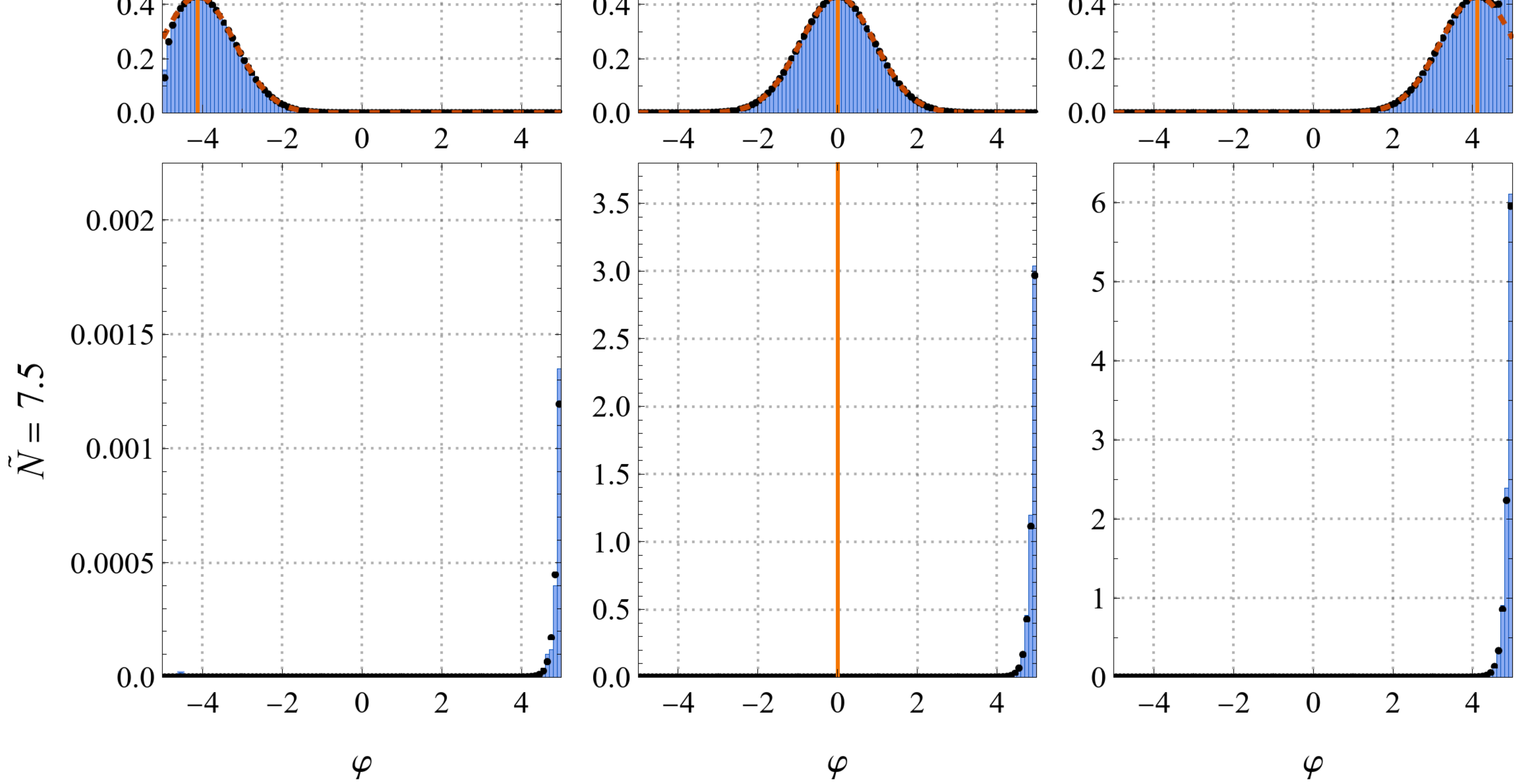
$$\frac{1}{2} \sigma^2(\phi) \partial_\phi^2 \bar{u}_n(\phi) + \mu(\phi) \partial_\phi \bar{u}_n(\phi) = -\lambda_n \bar{u}_n(\phi)$$



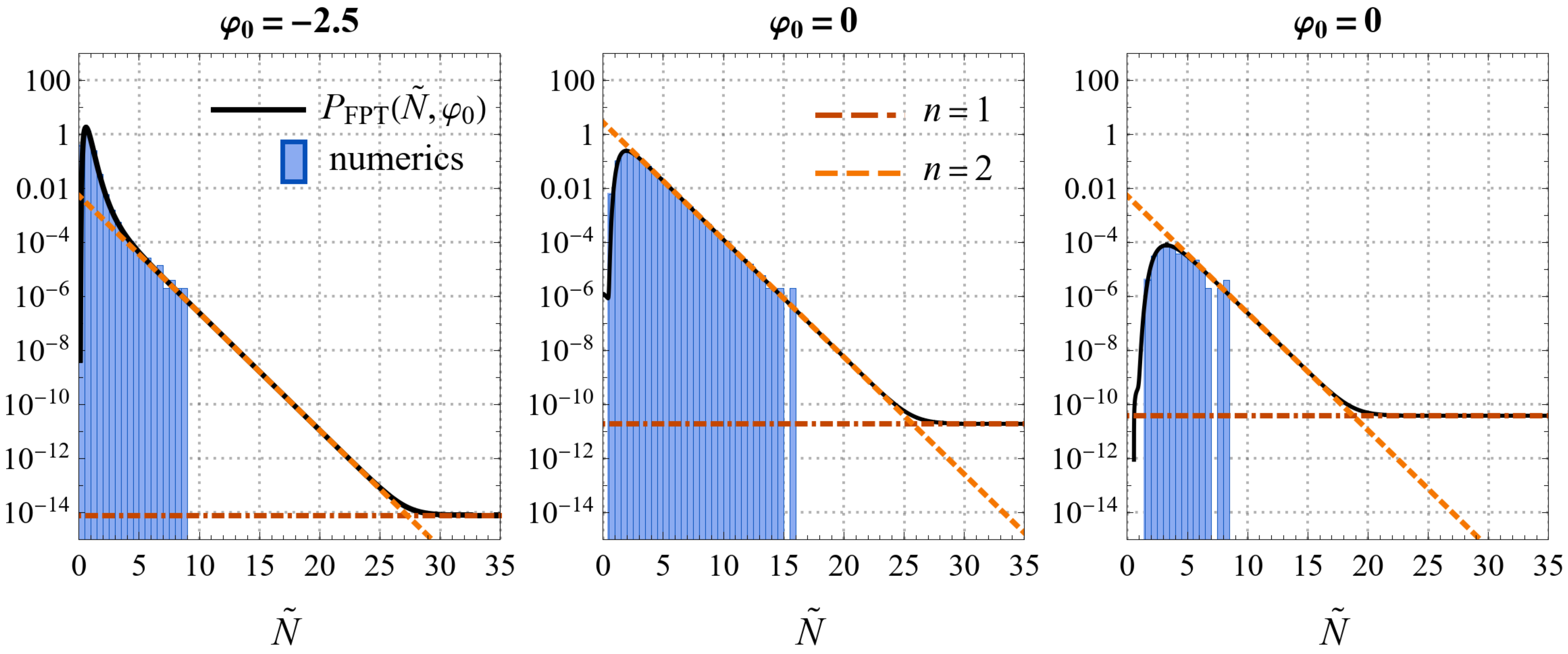
$\varphi_0 = -2.5$
$\varphi_0 = 0$
$\varphi_0 = 2.5$
$\tilde{N} = 0.05$
$P(\varphi, \tilde{N} \,|\, \varphi_0)$
numerics
spectral
Gaussian
classical



classical
$\tilde{N} = 0.5$



0.4
0.2
0.0
−4
−2
0
2
4
$\tilde{N} = 7.5$
0.002
0.0015
0.001
0.0005
0.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
6
5
3
1
φ



$\varphi_0 = -2.5$
$\varphi_0 = 0$
$\varphi_0 = 0$
$P_{\rm FPT}(\tilde{N}, \varphi_0)$
numerics
$n = 1$
$n = 2$
100
1
0.01
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
0 5 10 15 20 25 30 35
$\tilde{N}$

Hilltop inflation: ΔN

Coarse-grained curvature perturbation:

$$\Delta N_c = N_c + \bar{N}_c(\phi_c) - \bar{N}$$

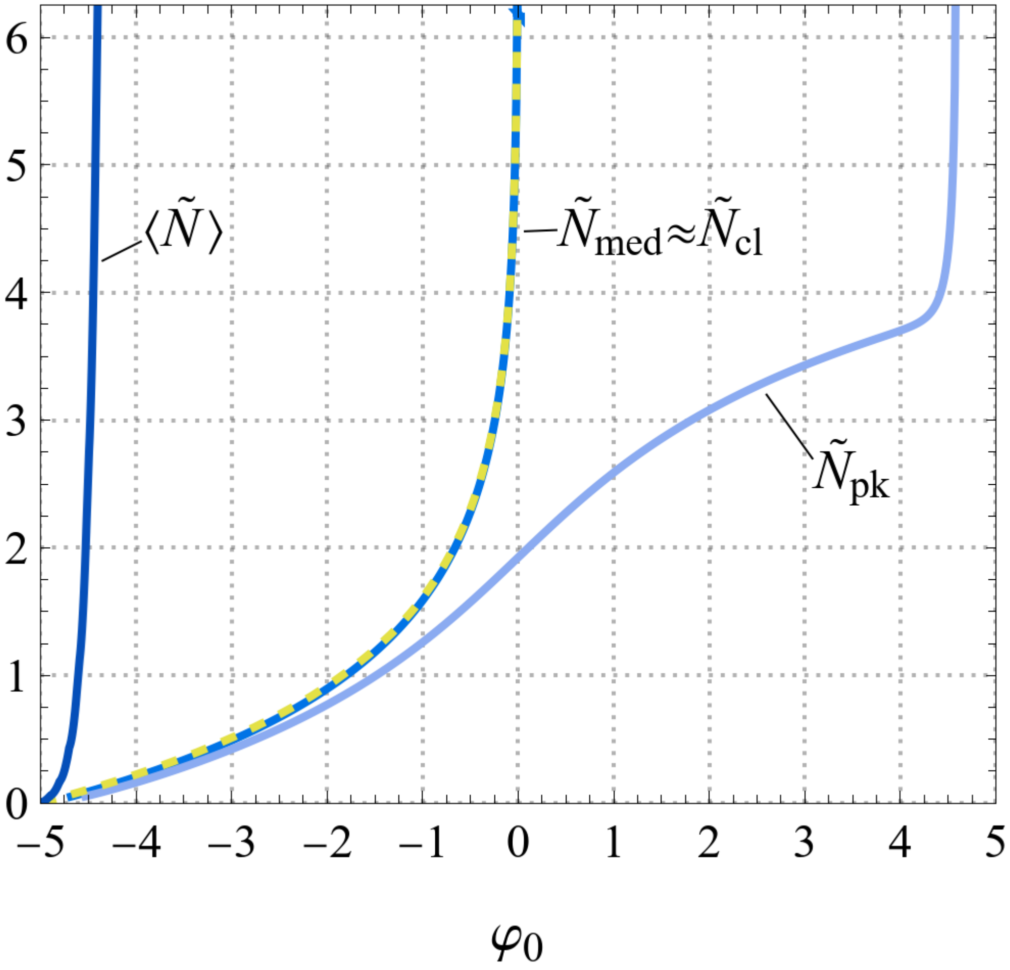
Coarse-graining scale

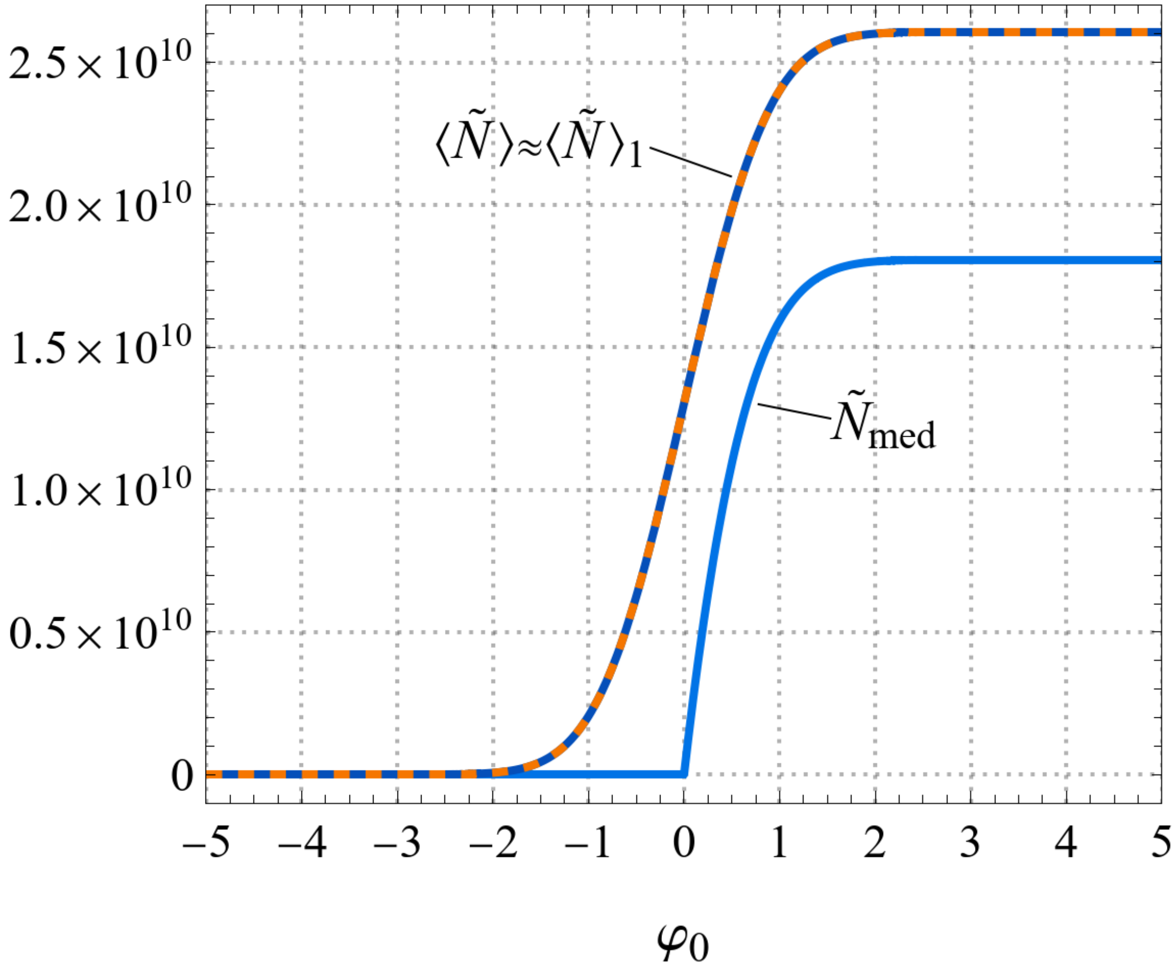
Stochastic

$$P(\Delta N_c) = P(\phi_c, N_c)\left|\frac{\mathrm{d}\phi_c}{\mathrm{d}\bar{N}_c}\right|$$

Background?

Use median!



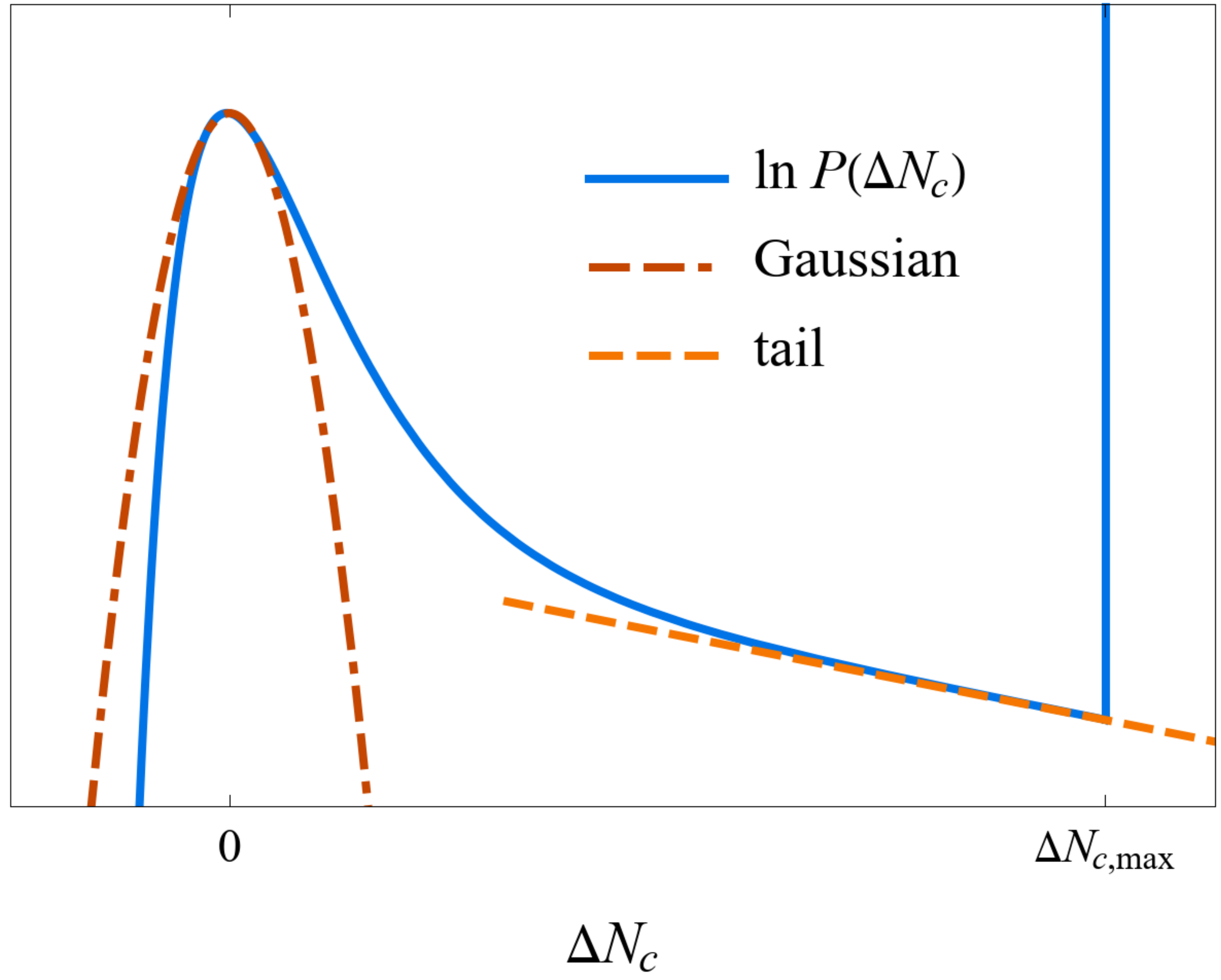


Mean fails as background!

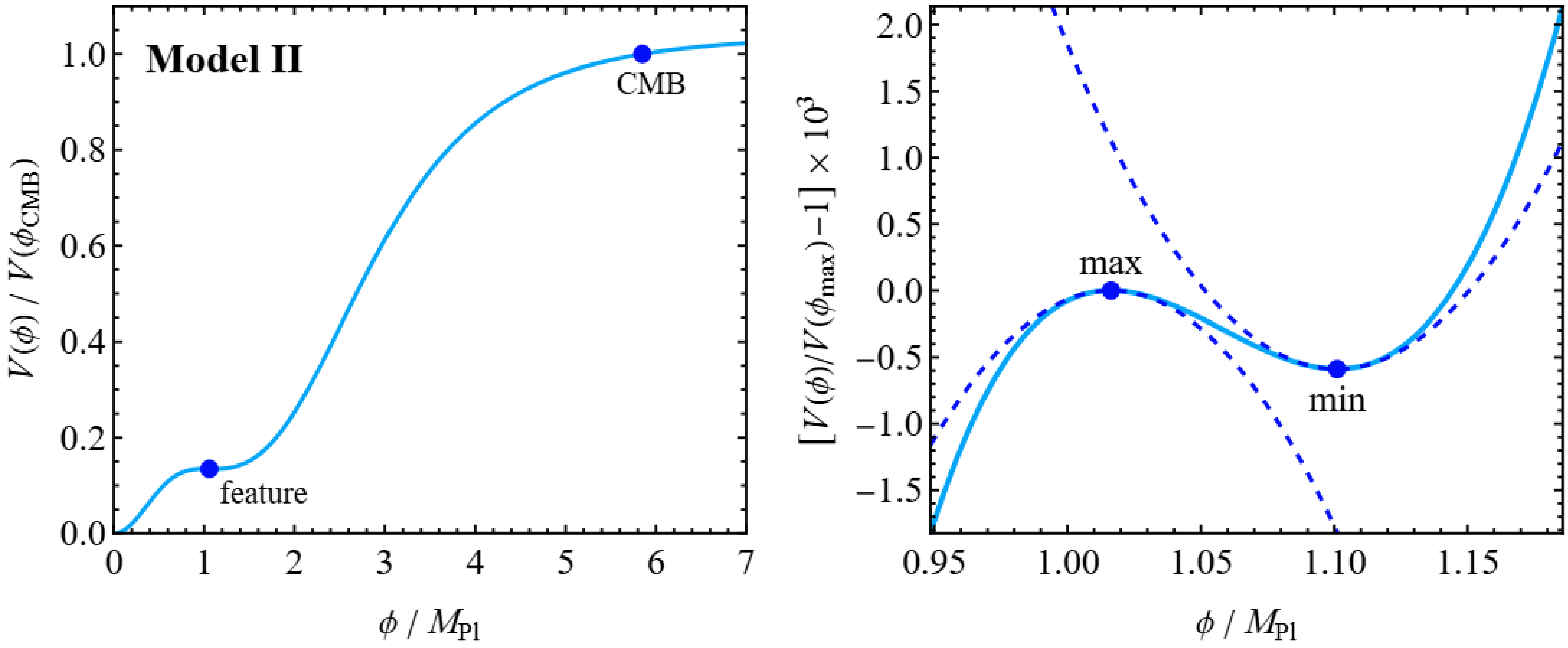
Use median?

Beyond hilltop?

Hilltop inflation: ΔN



PBH models



Model II
CMB
feature
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
7
$V(\phi)$ / $V(\phi_{\mathrm{CMB}})$
ϕ / M_{Pl}
max
min
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0.95
1.00
1.05
1.10
1.15
$[V(\phi)/V(\phi_{\mathrm{max}})-1] \times 10^3$
ϕ / M_{Pl}

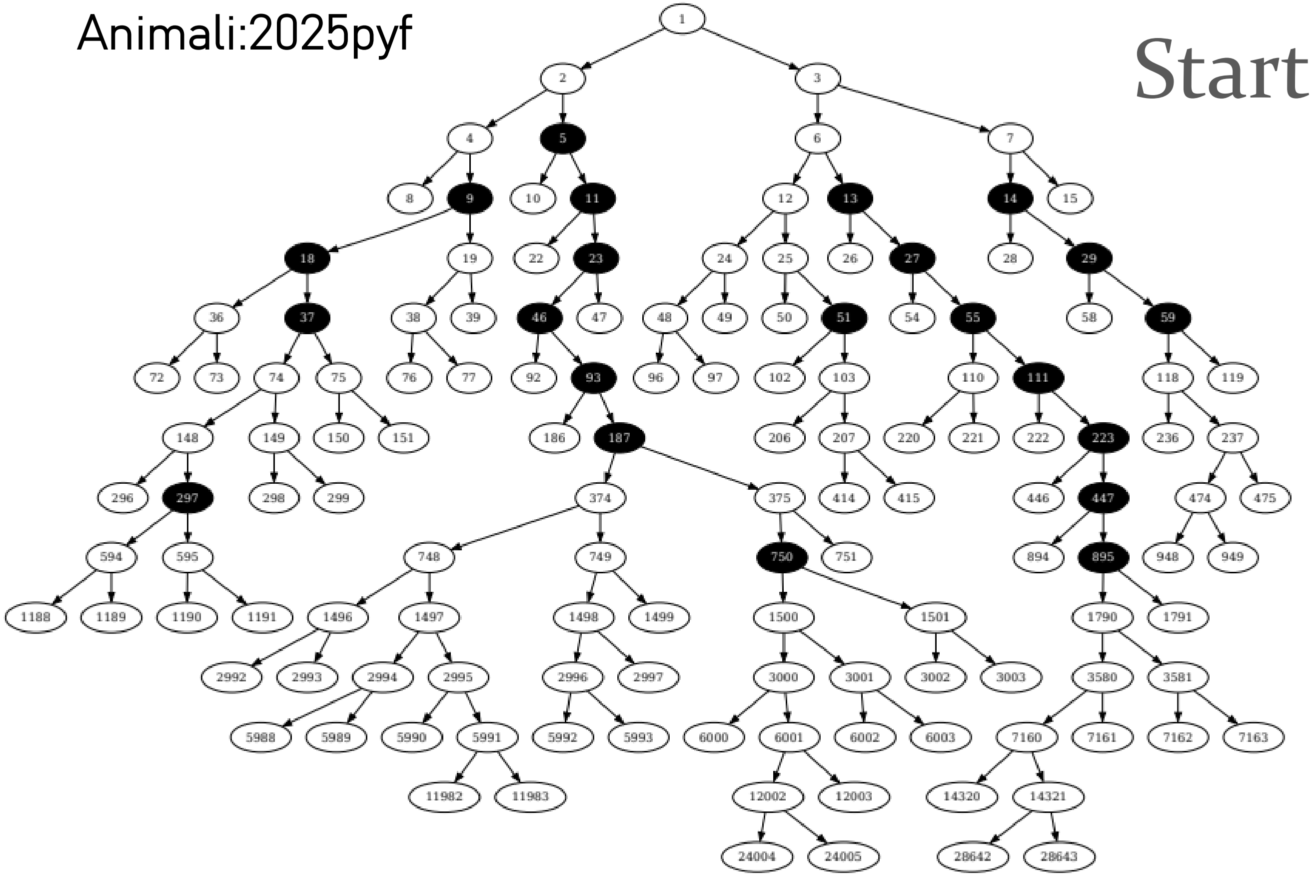
Eternal inflation

$$\lim_{N\to\infty} \langle V \rangle = \lim_{N\to\infty} \int_{\phi_a}^{\phi_r} \mathrm{d}\phi \; P(\phi, N) e^{3N} > 0$$

$$\iff \quad \lambda_1 \leq 3$$

Stochastic trees

Animali:2025pyf



Start from 1 Hubble patch

Once volume doubled: split in two

Spatial distribution? Clustering?

Lattice simulations

Mizuguchi:2024kbl

Stochastic grid

RUP-24-10

Prepared for submission to JCAP

STOLAS: STOchastic LAttice Simulation of cosmic inflation

STOchastic LAttice Simulation

[astro-ph.CO] 20 Dec 2024

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Launay:2024qsm

Combining stochastics and numerical relativity

Stochastic Inflation in General Relativity

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[2]*School of Mathematics, Statistics and Physics, Newcastle University, Newcastle upon Tyne, NE1 7RU, United Kingdom*

We provide a formulation of Stochastic Inflation in full general relativity that goes beyond the slow-roll and separate universe approximations. We show how gauge invariant Langevin source terms can be obtained for the complete set of Einstein equations in their ADM formulation by providing a recipe for coarse-graining the spacetime in any small gauge. These stochastic source terms are defined in terms of the only dynamical scalar degree of freedom in single-field inflation and all depend simply on the first two time derivatives of the coarse-graining window function, on the gauge-invariant mode functions that satisfy the Mukhanov-Sasaki evolution equation, and on the slow-roll parameters. It is shown that this reasoning can also be applied to include gravitons as stochastic sources, thus enabling the study of all relevant degrees of freedom of general relativity for inflation. We validate the efficacy of these Langevin dynamics directly using an example in uniform field gauge, obtaining the stochastic e-fold number in the long wavelength limit without the need for a first-passage-time analysis. As well as investigating the most commonly used gauges in cosmological perturbation theory, we also derive stochastic source terms for the coarse-grained BSSN formulation of Einstein's equations, which enables a well-posed implementation for 3+1 numerical relativity simulations.

30v2 [gr-qc] 7 May 2024

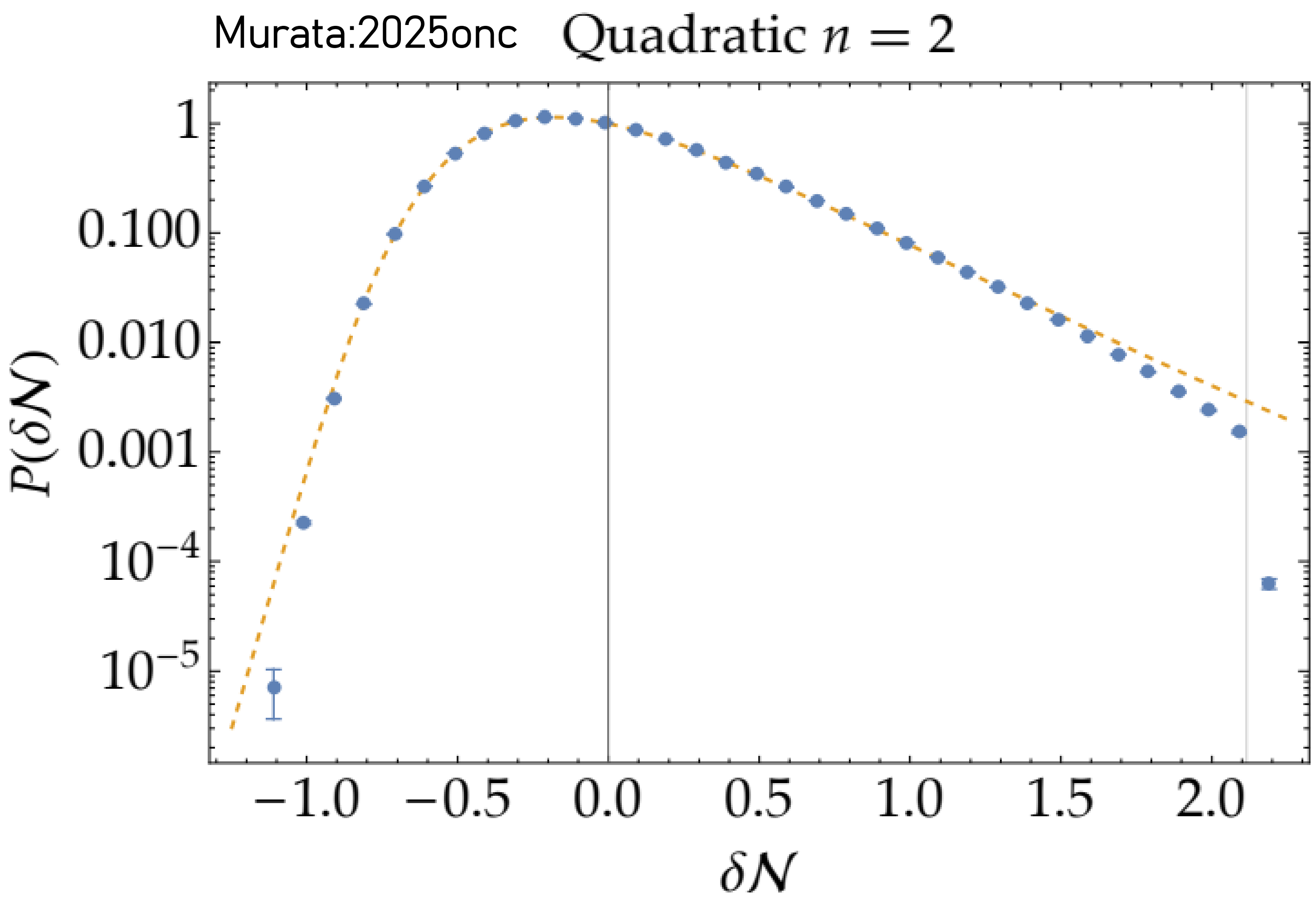
I. INTRODUCTION

Inflation theory was postulated more than 40 years ago as an explanation for the apparently fine-tuned initial conditions of the Hot Big Bang [1–3]. The proposal gained traction as it also offers a natural mechanism for generating the initial density inhomogeneities [4–9] which in later stages of cosmic history led to the formation of cosmic structure via gravitational instability. These den-

efforts [13–18], there are regimes where predictions can still be made by using techniques from Quantum Field Theory on Curved Spacetime (QFTCS) [15] or by constructing Effective Field Theories (EFTs), which have made continuous advancements in cosmology, inspired by the latter's success in flat space [19]. Abandoning pretenses of completeness, an EFT establishes a region of validity, normally bounded by ultraviolet (UV) and/or infrared (IR) cutoffs, and the narrative of theoretical

Multi-field inflation

Murata:2025onc



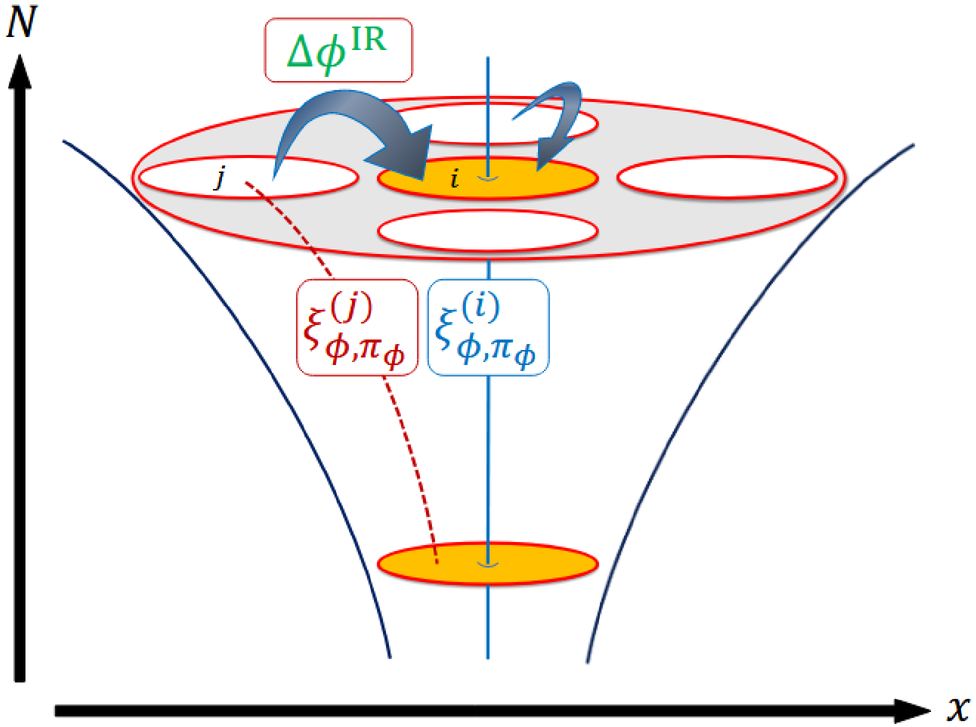
Hybrid inflation model

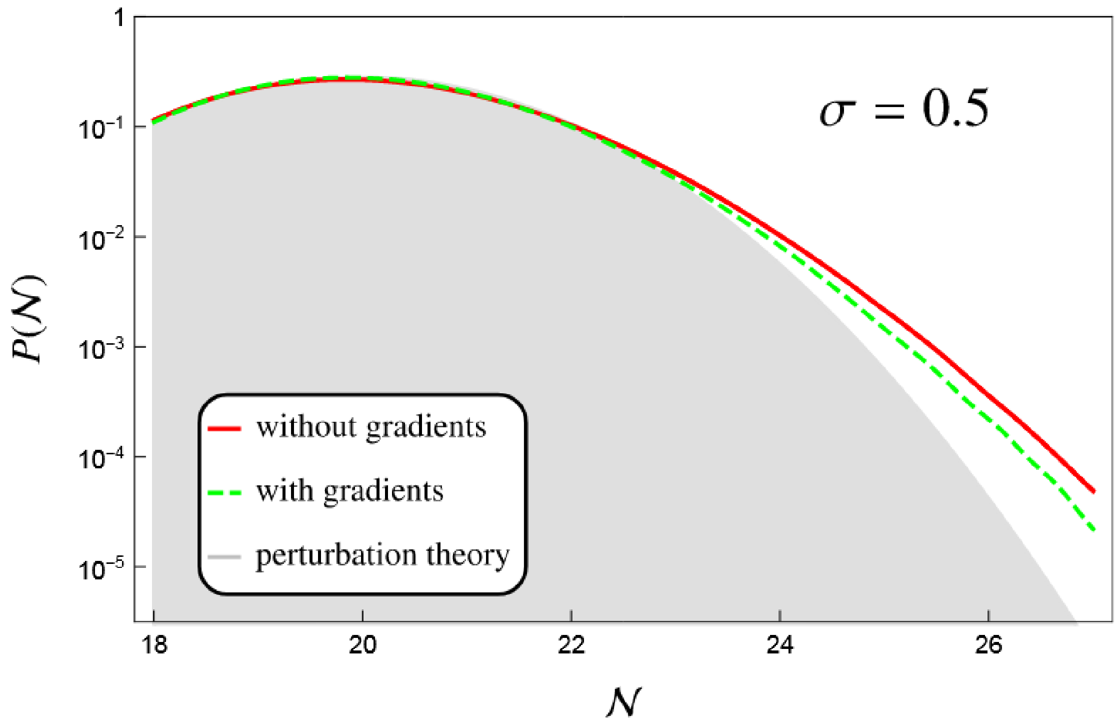
Upper bound on curvature perturbation

Gradient corrections

Briaud:2025ayt

Go beyond leading order in gradient expansion





Colored noise

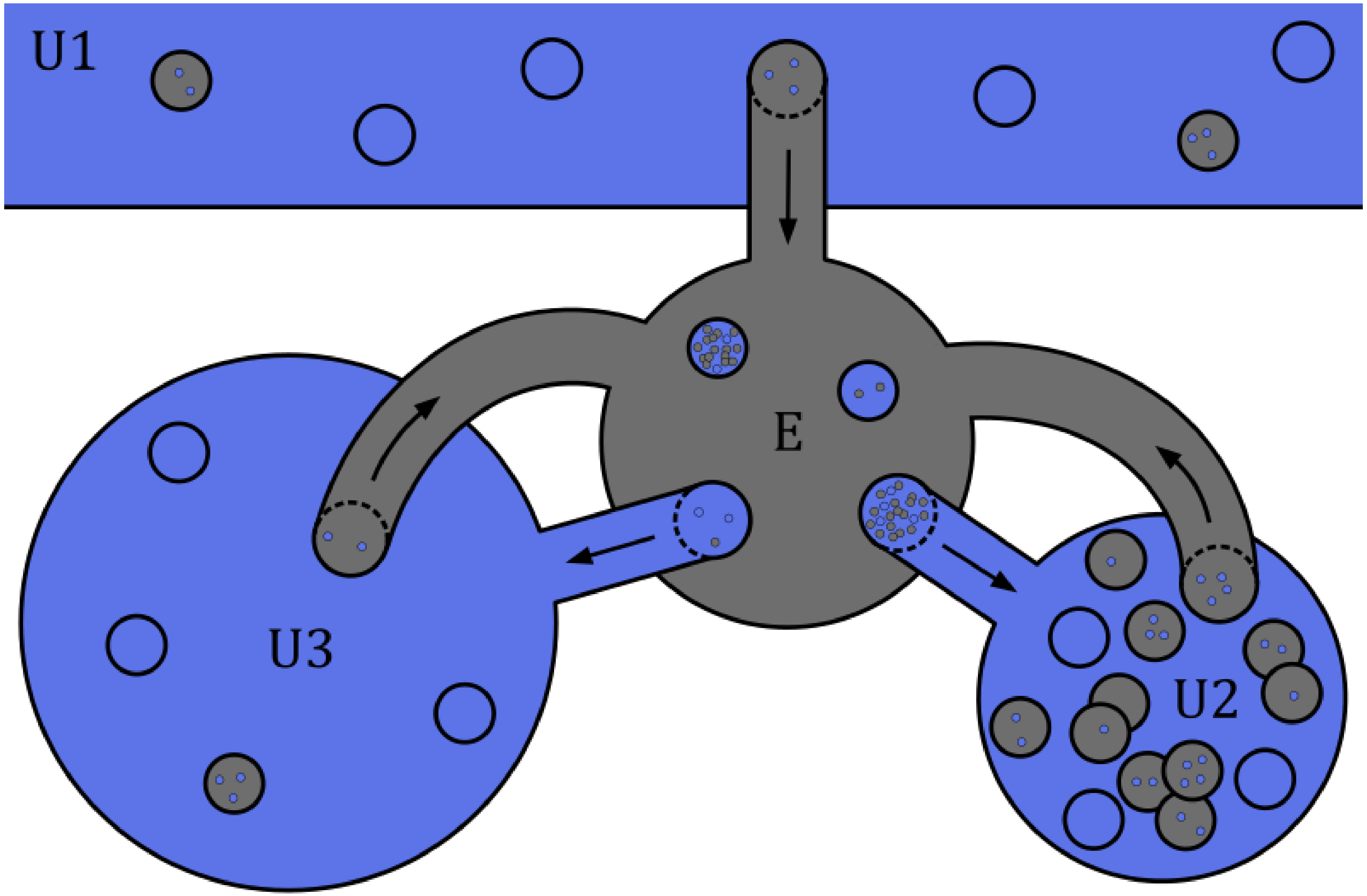
Exponential tails: a pullback effect

Summary

Extreme inflationary perturbations:
an active research field

Stochastic inflation well-suited for
studying largest perturbations

Characterising perturbations is tricky:
how to define background?


U1
E
U3
U2